Adaptive Optics Modeling and Simulation

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2009 CfAO Summer School
Lecture Topics

- Introductory remarks on adaptive optics
- Wave optics simulations
- Rytov theory
- Turbulence monitoring
- PSF estimation
- Differential Astrometry
Introductory Remarks
Wave Propagation through Turbulence

Science Target

Guide Star

Science Target

Turbulence

Telescope Aperture

Turbulence

Telescope Aperture

Tip Tilt Guide Star

Laser Guide Star

Science Target

Telescope Aperture

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The Adaptive Optics Backend

Deformable Mirror → Beamsplitter → Real Time Controller → Wavefront Sensor → From Guide Star → Science Camera
Wave Optics Simulations
Atmospheric Turbulence

Science Target

Wavefront Phase

Telescope Aperture

Turbulence

1.25 μm PSF

1.65 μm PSF

2.2 μm PSF

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Elements of a Wave Optics Simulation

- Generation of random phase screens to form a turbulence profile.
- Wave propagation through turbulence.
- Simulation of the wavefront sensor
- Reconstruction and control
- Simulation of the deformable mirror
- Formation of the science image
Phase Screens

Two dimensional phase screens are generated in the spatial frequency domain by filtering gaussian random noise with the power spectrum.

The resulting two dimensional arrays are Fourier transformed to produce a physical phase screen.

Periodic or aperiodic phase screens may be generated. The choice depends on the application.
Phase Screen Dynamics
Split-Step Wave Propagation Through Turbulence

Wavefront Amplitude

Wavefront Phase

Add phase errors
Propagate
Add phase errors
Propagate
Add phase errors

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An Example: Anisoplanatism

Propagation Geometry

Differential wavefront phase vs. field location
An Example: Scintillation

5 meters

z = 0.1  \hspace{1cm} 0.5 \text{ um}  \hspace{1cm} 1 \text{ um}  \hspace{1cm} 2 \text{ um}

\[\leftarrow 5 \text{ meters} \rightarrow\]
Pupils

Annulus

Spidered Annulus

Tiled Hexagons

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Pupil and Focal Plane Imagery in the Absence of Turbulence

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Shack-Hartmann Wavefront Sensors
Shack-Hartmann Wavefront Sensors Imaging Turbulence
Deformable Mirrors

The simplest model consists of pyramid actuator influence functions.

More realistic models for influence functions may be pursued using the plate equation.
Deformable Mirror Surface Simulation

0.0000 s
Wavefront Reconstruction

\[
\begin{bmatrix}
\text{Centroids}
\end{bmatrix} = \Gamma \begin{bmatrix}
\text{Actuator Commands}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{Actuator Commands}
\end{bmatrix} = \Gamma^+ \begin{bmatrix}
\text{Centroids}
\end{bmatrix}
\]
The Resulting DM Surface

Distorted wavefront entering the Shack Hartmann sensor

The correction as sensed by the Shack Hartmann, reconstructed and placed onto the DM

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An AO Simulation

Aberrated Phase

TTM Surface

DM Surface

Residual Phase

PSF

DL PSF

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Simulation Parameters

- Turbulence profile and power spectra
- Guide star/science target geometry
- Diffractive/geometric propagation
- Sensing and detection wavelengths
- Phase screen sampling
- Aperture diameter/geometry
- Style of wavefront sensing and order of correction
- Pupil / lenslet array / deformable mirror registration
- Reconstruction and control algorithms
- Focal plane resolution
Adaptive Optics PSF for A 30m Segmented Telescope

Diffraction Limited PSF
AO Compensated PSF
Misregistration Tolerance
Control Law Analysis

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Compute Time Requirements on a 2.4 GHz Pentium

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Parallelization of Wave Optics Simulations

**Memory and CPU resources**
- Duration of simulation
- Number of guide stars and science targets
- Level of fidelity

**Computational architecture**
- Symmetric multiprocessor
- Cluster

**Simulation Output**
- Data load
- Results
Rytov Theory and Turbulence Monitoring
Rytov Approximation

The Rytov approximation provides expressions for the two-point phase and log amplitude correlation function between rays traveling in different directions through turbulence.

\[
\langle \phi_a (\vec{r}_1) \phi_b (\vec{r}_2) \rangle
\]

\[
\langle \chi_a (\vec{r}_1) \chi_b (\vec{r}_2) \rangle
\]
Aperture Averaged Rytov Expressions for Plane Waves

\[
\begin{pmatrix}
\langle \phi_a \rangle \\
\langle \chi_a \rangle
\end{pmatrix}
= \begin{pmatrix}
Yk_0^2 \int_0^L dz \ C_n^2(z) \int d\vec{k} \ f(\vec{k}) \\
\end{pmatrix}
\begin{bmatrix}
\cos^2 \left( \frac{\kappa^2 z}{2k_o} \right) \\
\sin^2 \left( \frac{\kappa^2 z}{2k_o} \right)
\end{bmatrix}
F(\vec{k})
\]

\[Y = 0.207379\]

Sasiela 2007
Multi-Aperture Scintillation Sensor
MASS Integrands in the Rytov Approximation
MASS Weighting Functions

\[
\langle \chi^2 \rangle = \int_0^L dz \ C_n^2(z) \ \Gamma(z)
\]

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The Inverse Problem

\[
\begin{bmatrix}
\langle \chi_A^2 \rangle \\
\langle \chi_B^2 \rangle \\
\langle \chi_C^2 \rangle \\
\langle \chi_D^2 \rangle \\
\langle \chi_{AB}^2 \rangle \\
\langle \chi_{AC}^2 \rangle \\
\langle \chi_{AD}^2 \rangle \\
\langle \chi_{BC}^2 \rangle \\
\langle \chi_{BD}^2 \rangle \\
\langle \chi_{CD}^2 \rangle \\
\end{bmatrix} = \mathbf{\tilde{A}}
\begin{bmatrix}
C_n^2 (16 \text{ km}) \, dz \\
C_n^2 (8 \text{ km}) \, dz \\
C_n^2 (4 \text{ km}) \, dz \\
C_n^2 (2 \text{ km}) \, dz \\
C_n^2 (1 \text{ km}) \, dz \\
C_n^2 (.5 \text{ km}) \, dz \\
\end{bmatrix}
\]
The DIMM/MASS Instrument

Pupil plane mask

DIMM apertures

Kornilov et al., MNRAS 2007
DIMM/MASS Results

http://odata1.palomar.caltech.edu/massdimm/

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Seeing and $r_0$ Variability

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Seeing and $r_0$ Statistics

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Pupil-Dependent Rytov Expressions for Plane Waves

There are many circumstances in which we would like to employ expressions that are not aperture averaged.

\[
\left\langle \left[ \phi(\vec{r}_1) \right]^2 \right\rangle
\]

\[
D_{\phi}(\vec{r}_1, \vec{r}_2) = \left\langle \left[ \phi(\vec{r}_1) - \phi(\vec{r}_2) \right]^2 \right\rangle
\]

\[
OTF(\vec{r}) = \iint d\vec{s} \exp \left\{ -\frac{1}{2} D_{\phi}(\vec{s}, \vec{r} + \vec{s}) \right\}
\]

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The Two-Point Correlation Function for Plane Waves

\[ \langle \phi_b(\vec{r}_1) \phi_a(\vec{r}_2) \rangle = \Xi k^2 D^{5/3} \int_0^\infty dz C_n^2(z) \]

\[ \left\{ G_1 \left( \frac{2 \vec{r}_1}{D} + \frac{2z\vec{\theta}_{ab}}{D} \right) + G_1 \left( \frac{2 \vec{r}_2}{D} - \frac{2z\vec{\theta}_{ab}}{D} \right) \right\} - \]

\[ \left\{ 2(\vec{r}_1 - \vec{r}_2) \right\} \frac{5/3}{D} + \frac{2z\vec{\theta}_{ab}}{D} \]  

\[ - G_2 \left( \frac{2z\vec{\theta}_{ab}}{D} \right) \]

\[ \Xi = .4589 \}

\[ \langle \phi_b(\vec{r}_1) \phi_a(\vec{r}_2) \rangle = F(k, D, C_n^2(z), \vec{\theta}_{ab}, \xi, \vec{r}_1, \vec{r}_2) \]

NOTE:
Phase Structure Function for Uncompensated Turbulence

\[ D_\phi(\vec{r}_1, \vec{r}_2) = \left\langle [\phi(\vec{r}_1)]^2 \right\rangle + \left\langle [\phi(\vec{r}_2)]^2 \right\rangle - 2\left\langle [\phi(\vec{r}_1)\phi(\vec{r}_2)]^2 \right\rangle \]

\[ = \Xi k^2 (\vec{r}_1 - \vec{r}_2)^{5/3} \int_0^\infty dz C_n^2(z) \]

\[ r_o^{-5/3} = 2^{8/3} \frac{\Xi}{\Lambda} k_o^2 \int dz C_n^2(z) \quad \Lambda = 6.8838 \]

\[ D_\phi(\vec{r}_1, \vec{r}_2) = \Lambda \left( \frac{(\vec{r}_1 - \vec{r}_2)}{r_o} \right)^{5/3} \]
Tilt Removed Phase Variance on an Aperture

\[
\left\langle \left[ \phi(\vec{r}) - \phi^T(\vec{r}) \right] \right\rangle
\]
## Readily Available Simulations

<table>
<thead>
<tr>
<th>Name</th>
<th>Lang</th>
<th>Methods</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAOS</td>
<td>IDL</td>
<td>Wave Optics</td>
<td>www-luan.unice.fr/caos/</td>
</tr>
<tr>
<td>ESO’s sim</td>
<td>C++</td>
<td>Wave Optics</td>
<td>Contact: Miska Le Lou</td>
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<td>Paola</td>
<td>IDL</td>
<td>Wave Optics</td>
<td>cfao.ucolick.org/software/paola</td>
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<td>YAO</td>
<td>Yorick</td>
<td>Wave Optics</td>
<td><a href="http://www.maumae.net/yao/">www.maumae.net/yao/</a></td>
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<td>LAOS</td>
<td>Matlab</td>
<td>Wave Optics</td>
<td>Contact: Luc Gilles</td>
</tr>
<tr>
<td>Cibola</td>
<td>Matlab</td>
<td>Rytov</td>
<td>cfao.ucolick.org/software/cibola</td>
</tr>
<tr>
<td>Arroyo</td>
<td>C++</td>
<td>Wave Optics and Rytov</td>
<td>eraserhead.caltech.edu/arroyo.</td>
</tr>
<tr>
<td>SciAO</td>
<td>Scios</td>
<td>Wave Optics</td>
<td>sciao.sourceforge.net/</td>
</tr>
</tbody>
</table>
PSF Estimation
The Structure Function for NGS AO

Residual phase

\[ \psi(\vec{r}) = \phi_b(\vec{r}) - \phi_a(\vec{r}) + \tilde{\phi}_a(\vec{r}) \]

Structure function of the residual phase

\[ D_\psi(\vec{r}_1, \vec{r}_2) = D_{apl}(\vec{r}_1, \vec{r}_2) + D_{\phi}(\vec{r}_1, \vec{r}_2) \]

May be computed using the 2 pt correlation function:

\[ \left\langle (\phi_b(\vec{r}_1) - \phi_a(\vec{r}_1))(\phi_b(\vec{r}_2) - \phi_a(\vec{r}_2)) \right\rangle \]
The NGS AO Structure Function

\[
D_{\psi}(\vec{r}_1, \vec{r}_2) = D_{apl}(\vec{r}_1, \vec{r}_2) + D_{\phi}(\vec{r}_1, \vec{r}_2)
\]

\[
D_{apl}(\vec{r}_1, \vec{r}_2) = 2^{8/3} \Xi k^2 \int_0^\infty dz C_n^2(z)
\]

\[
\left\{ \begin{array}{l}
\left| z\theta_{ab} \right|^{5/3} + 2\left| \vec{r}_1 - \vec{r}_2 \right|^{5/3} - \left| \vec{r}_1 - \vec{r}_2 + z\theta_{ab} \right|^{5/3} - \left| \vec{r}_1 - \vec{r}_2 - z\theta_{ab} \right|^{5/3}
\end{array} \right.
\]

A bonus! The NGS structure function is stationary.

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The NGS OTF and PSF

\[ OTF(\vec{r}) = \iint d\vec{s} \exp \left\{ -\frac{1}{2} D_{\psi}(s, \vec{r} + s) \right\} \]

\[ = \exp \left\{ -\frac{1}{2} D_{apl}(\vec{r}) \right\} \iint d\vec{s} \exp \left\{ -\frac{1}{2} D_{\phi}(s, \vec{r} + s) \right\} \]

Anisoplanatic Transfer Function

Guide Star OTF

Example OTF and PSF for a perfect AO system
Long Exposure PSF of a Perfect AO System
Experimental Validation

A 21” binary

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Differential Astrometry
Differential Tilt Jitter

\[
\begin{bmatrix}
\sigma_{\parallel}^2 \\
\sigma_{\perp}^2
\end{bmatrix}
= 2.67 \, \frac{\mu_2}{D^{1/3}} \left( \frac{\theta}{D^2} \right)^3 + \ldots
\]
Differential Astrometry

\[ P(d) = \frac{1}{\sqrt{2\pi \det \Sigma_d}} \exp \left\{ \frac{1}{2} \left[ d - \bar{d} \right]^T \Sigma_d^{-1} \left[ d - \bar{d} \right] \right\} \]

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Grid Astrometry

We would like to establish the location of the target star with respect to the grid of reference stars that lie in the field of view. To do so, we would like to average over the measured separation vectors between the target and reference stars. The question is how to do so in a way that minimizes the uncertainty in the resulting measurement.

\[ \vec{d}_1 \]

\[ \vec{d}_2 \]
Optimal Estimation

\[ \delta = Wd \]

\[ \Sigma_\delta = W^T \Sigma_d W \]
Improvement With Number of Reference Stars

![Graphs showing improvement in astrometric precision with number of reference stars.](image-url)
Three Observational Epochs of M5
Scaling Laws With Telescope Diameter
Lecture Summary
Topical Review

- Wave optics simulations and their role in the design and tolerancing of adaptive optics systems.
- Use of the Rytov theory in estimation of the turbulence profile from variance measurements.
- Use of the Rytov theory in modeling variances, structure functions and optical transfer functions on the aperture.
- Modeling anisoplanatic errors for PSF estimation and differential astrometry.